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THE LAST EIGHT MINUTES OF A PRIMORDIAL BLACK HOLE

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Abstract

About eight minutes before a black hole expires it has a decreasing mass of 10^{10} g, an increasing temperature of 1 TeV, and an increasing luminosity of 7×10^{27} erg/s. I show that such a black hole is surrounded by a quasi-stationary shell of matter undergoing radial hydrodynamic expansion. The inner radius of this shell is bounded by ten times the Schwarzschild radius of 1.6×10^{-5} fm and has a temperature about one-tenth that of the black hole. The outer radius, as defined by the photosphere, is about 1000 fm, has a local temperature of 100 keV, and is moving with a Lorentz gamma factor of 10^7. Most of the emitted radiation is in photons with small amounts in gravitons and neutrinos. I calculate the instantaneous photon spectrum and then integrate it over the last eight minutes to obtain the energy distribution $dN_\gamma/dE = 4\pi m_{\rm P}^2/15E^3$ for $E >$ several TeV .

Hawking radiation from black holes [1] is of fundamental interest because it relies on the application of relativistic quantum field theory in the presence of the strong field limit of gravity, a so far unique situation. It is also of great interest because of the temperatures involved. A black hole with mass M radiates thermally with a temperature

$$T_{\rm h} = \frac{m_{\rm P}^2}{8\pi M} \tag{1}$$

where $m_{\rm P} = G^{-1/2} = 1.22 \times 10^{19}$ GeV is the Planck mass. (I use units in which $\hbar = c = k_{\rm B} = 1$.) In order for the black hole to evaporate it must have a temperature greater than that of the present-day black-body radiation of the universe of 2.7 K = 2.3×10^{-4} eV. This implies that M must be less than 0.8% of the mass of the Earth, hence the black hole must have been formed primordially and not from a stellar collapse. The black hole temperature eventually goes to infinity as its mass goes to zero, although once $T_{\rm h}$ becomes comparable to the Planck mass the semi-classical calculation breaks down and the regime of full quantum gravity is entered. Only in two other situations are such enormous temperatures achievable: In the early universe (T similarly asymptotically high) and in central collisions of heavy nuclei like gold or lead ($T = 500$ MeV is expected at the RHIC (Relativistic Heavy Ion Collider) just completed at Brookhaven National Laboratory and $T = 1$ GeV is expected at the LHC (Large Hadron Collider) at CERN to be completed in 2005). The spontaneously broken chiral symmetry of QCD gets restored in a phase transition/rapid crossover at a temperature around 160 MeV while the spontaneously broken gauge symmetry in the electroweak sector of the standard model gets restored in a phase transition/rapid crossover at a temperature around 100 GeV. The fact that temperatures of the latter order of magnitude will never be achieved in a terrestrial experiment motivates me here to study the fate of primordial black holes during the final minutes of their lives when their temperatures have risen to 100 GeV and above. The fact that primordial black holes have not yet been observed [2] does not deter me in the least.

When $T_{\rm h} \ll m_{\rm e}$ (electron mass) only photons, gravitons, and neutrinos will be created with any significant probability. These particles will not interact with each other but will be emitted into the surrounding space with the speed of light. Even when $T_{\rm h} \approx m_{\rm e}$ the Thomson cross section is too small to allow the photons to scatter very frequently in the rarified electron-positron plasma around the black hole. This may change when $T_{\rm h} \approx 80-100$ MeV when muons and charged pions are created in abundance. At somewhat higher temperatures hadrons are copiously produced and local thermal equilibrium may be achieved, although exactly how is an unsettled issue. Are hadrons emitted directly by the black hole? If so, they will be quite abundant at temperatures of order 150 MeV because their mass spectrum rises exponentially (Hagedorn growth as seen in the Particle Data Tables [3]). Because they are so massive they move nonrelativistically and may form a very dense equilibrated gas around the black hole. But hadrons are composites of quarks and gluons, so perhaps quarks and gluon jets are emitted instead? These jets must

decay into the observable hadrons on a typical length scale of 1 fm and a typical time scale of 1 fm/c. Once the hadrons appear they may form an equilibrated gas around the black hole just as if they had been produced directly albeit with some time delay. One can find arguments both for [4] and against [5] thermal equilibrium being maintained by the strongly interacting hadrons outside the Scharzschild radius. Certainly this is a very difficult and open problem in quantum statistical mechanics. Fortunately the answer is unambiguous for black hole temperatures greater than the electroweak scale of 100 GeV, as we shall see.

I assume that a primordial black hole is surrounded by a shell of hydrodynamically expanding matter in local thermal equilibrium when $T_{\rm h} > 100$ GeV. This assumption will be shown to be self-consistent. A detailed description of how this situation comes to be is a difficult problem as discussed above and is not addressed in this paper.

The relativistic hydrodynamic equations describing an adiabatic, steady-state, spherically symmetric flow with no net baryon number or electric charge and neglecting gravity were derived by Flammang and Thorne [6]. The equations may be cast in the following form.

$$4\pi r^2 v\gamma^2 T s(T) \;=\; L_{\rm in} = \text{ luminosity} \tag{2}$$

$$\gamma T \;=\; \text{constant} \tag{3}$$

Here r is the radial coordinate, v is the local flow velocity with the associated Lorentz factor γ, T is the local temperature, and $s(T)$ is the entropy density of the fluid. The $L_{\rm in}$ is the luminosity flowing into the fluid. It is generally less than the total luminosity of the black hole. For example, gravitons will escape without scattering from the matter and so will not participate in the hydrodyamic flow. These equations were derived in the context of accretion but, of course, they apply equally well to expansion. They were used to describe gamma-ray bursts from neutron stars by Paczyǹski [7]. Given the luminosity these equations allow one to find the functions $v(r)$ and $T(r)$ algebraically without solving any differential equations.

A black hole has a Schwarzschild radius $R_{\rm h} = 2M/m_{\rm P}^2 = 1/4\pi T$. Note that $\pi T_{\rm h} \cdot 2R_{\rm h} = 1/2$. Roughly, the average thermal momentum of a massless particle times the diameter of the black hole is 1/2. This is just a manifestation of the uncertainty principle applied to the creation of an excitation in a confined region of space. The luminosity is

$$L = -\frac{dM}{dt} = \alpha(M)\frac{m_{\rm P}^4}{M^2} = 64\pi^2\alpha(T_{\rm h})T_{\rm h}^2 \tag{4}$$

where $\alpha(M)$ is a function reflecting the species of particles available for creation in the gravitational field of the black hole. It is generally sufficient to consider only those particles with mass less than $T_{\rm h}$; more massive particles are exponentially suppressed by the Boltzmann factor. I use

$$\alpha = 2.011 \times 10^{-8}\left[4200 N_0 + 2035 N_{1/2} + 835 N_1 + 95 N_2\right] . \tag{5}$$

Here N_s is the net number of polarization degrees of freedom for all particles with spin s. The coefficients for spin 1/2, 1 and 2 were computed by Page [8] and for spin 0 by Sanchez [9]. In the standard model $N_0 = 4$ (Higgs), $N_{1/2} = 90$ (three generations of quarks and leptons), $N_1 = 24$ (SU(3)×SU(2)×U(1) gauge theory), and $N_2 = 2$ (gravitons). This assumes T_h is greater than the temperature for the electroweak gauge symmetry restoration [10]. Numerically $\alpha(T_\mathrm{h} > 100\,\mathrm{GeV}) = 4.43 \times 10^{-3}$.

The entropy of a black hole is given by the area formula.

$$S_\mathrm{h} = m_\mathrm{P}^2 \pi R_\mathrm{h}^2 = 4\pi \frac{M^2}{m_\mathrm{P}^2} = \frac{m_\mathrm{P}^2}{16\pi T_\mathrm{h}^2} \tag{6}$$

The entropy per unit time lost by the black hole $-dS_\mathrm{h}/dt$ is to be equated with that flowing through the matter, $4\pi r^2 v\gamma s$. Taking the ratio of this entropy rate to the luminosity allows us to determine the constant in the hydrodynamic equation.

$$\gamma T = T_\mathrm{h} \tag{7}$$

This result is independent of the function $\alpha(M)$.

The entropy density of weakly interacting massless particles is

$$s(T) = \frac{4\pi^2}{90} d(T) T^3 \tag{8}$$

where $d(T)$ is the number of bosonic degrees of freedom; fermions get counted with a weight of 7/8. Using this in eq. (2) it is found that v is a monotonically increasing function of r and that its minimum possible value is $v_\mathrm{min} = 1/\sqrt{3}$. This is a sensible value since that is the speed of sound of a weakly interacting gas of massless particles. The corresponding radius and fluid temperature are $r_\mathrm{min} = 1.43 R_\mathrm{h}$ and $T(r_\mathrm{min}) = 0.816 T_\mathrm{h}$. The former assumes that gravitons and neutrinos do not contribute to the entropy of the fluid. The actual radius at which the fluid may be considered as thermalized could very well be greater than r_min, but the fact that the latter is already greater than the Schwarzschild radius is eminently sensible.

The expanding fluid is in local thermal equilibrium when the mean free paths or thermalization lengths of the particles are less than the characteristic distance over which the temperature changes. This characteristic distance is $l = \gamma T |dr/dT|$ (the gamma takes into account the Lorentz transformation from the rest frame of the black hole to the rest frame of the fluid). When r is greater than a few times the Schwarzschild radius v is already approaching the speed of light and $\gamma \gg 1$. Then $r^2 T^2 = 360\alpha(T_\mathrm{h})/\pi d(T)$ and neglecting the slow variation of d with T (except near a first order phase transition)

$$\begin{aligned} l &= \frac{K T_\mathrm{h}}{T^2}, \\ K &= 6\sqrt{\frac{10}{\pi}\frac{\alpha(T_\mathrm{h})}{d(T)}}. \end{aligned} \tag{9}$$

Above 100 GeV the numerical value of K (leaving out gravitons and neutrinos) is 0.069.

In a relativistic plasma it is not sufficient to consider only 2-body reactions. Of prime importance in achieving and maintaining local thermal equilibrium are multi-body processes such as $2 \to 3$ and $3 \to 2$, etc. This has been well-known when calculating quark-gluon plasma formation and evolution in high energy heavy ion collisions [11] and has been emphasized by Heckler in the context of black hole evaporation [4]. This is a formidable task in the standard model with its 16 species of particles. Instead I require that the Debye screening length for each of the gauge groups in the standard model be less than l. The Debye screening length is the inverse of the Debye screening mass m_n^{D} where $n = 1, 2, 3$ for the gauge groups U(1), SU(2), SU(3). Generically $m_n^{\mathrm{D}} \propto g_n T$ where g_n is the gauge coupling constant and the coefficient of proportionality is essentially the square root of the number of charge carriers [12]. For example, for color SU(3)

$$m_3^{\mathrm{D}} = g_3 \sqrt{1 + N_{\mathrm{f}}/6}\, T \tag{10}$$

where N_{f} is the number of light quark flavors at the temperature T. The numerical values of the gauge couplings are: $g_1 = 0.344$, $g_2 = 0.637$, and $g_3 = 1.18$ (evaluated at the scale m_Z) [3]. So within a factor of about 2 I have $m^{\mathrm{D}} \approx T$. This estimate makes sense when compared to two other measures. The average energy of a massless particle is $3T$ and so m^{D} is about three times the thermal DeBroglie wavelength. On the other hand Carrington and Kapusta [13] calculated the mean time between two-body collisions in the standard model for temperatures greater than the electroweak symmetry restoration temperature in the process of calculating the viscosity in the relaxation time approximation. Averaged over all particle species in the standard model one may infer from that paper an average time of $3.7/T$. Taking into account multi-body reactions would decrease that by about a factor of two to four.

The conclusion to be drawn from the above is that local thermal equilibrium should be achieved when $1/T < l$ which translates to the condition $T < T_{\mathrm{h}}/15$. Once thermal equilibrium is achieved it is not lost as r increases because l increases as $1/T^2$ whereas the Debye screening length increases more slowly as $1/T$. The smallest radius at which local equilibrium is maintained is obtained from the condition on the temperature to be approximately $13R_{\mathrm{h}}$. This means that the thermalized matter surrounding the black hole is far enough away so that it doesn't affect the Hawking radiation whose calculation had assumed that the black hole radiated into a vacuum.

An amazing observation in experiments on central collisions of lead nuclei at a laboratory beam energy of 160 GeV per nucleon at the SPS at CERN is that the outgoing hadrons, such as nucleons, pions, and kaons, have spectra that appear to be very thermal [14]. In those collisions, quantities like local temperature, pressure, energy density, and particle density vary significantly over a length scale of several fm. For black hole temperatures greater than 100 GeV one may easily calculate l in that region of the expanding matter where it is in the hadronic phase, where the local temperature is on the order of

100 to 160 MeV, and find that it is much greater than several fm. Therefore thermal equilibrium is maintainted at least down to that range of temperatures.

Following Paczyǹski's analysis of neutron star gamma-ray bursters [7] let us assume that the matter maintains thermal equilibrium all the way down to a local temperature less than the electron mass. Gravitons and neutrinos have already escaped, leaving only photons and a rarified plasma of nonrelativistic electrons and positrons. The photosphere is defined as that radius where the optical depth is unity [15]. Interaction of the photons with the electrons and positrons proceeds through the Thomson cross section σ_{T}. Defining $n_\pm$ as the total density of non-relativistic electrons and positrons one has the Thomson mean free path for photons at temperature T.

$$\lambda_{\mathrm{T}} = \frac{1}{\sigma_{\mathrm{T}} n_\pm} = \frac{3}{8\pi} \frac{1}{\alpha_{\mathrm{EM}}^2} \frac{m_{\mathrm{e}}^2}{n_\pm} = 34.1\,\text{Å} \left(\frac{m_{\mathrm{e}}}{T}\right)^{3/2} \exp(m_{\mathrm{e}}/T) \tag{11}$$

Using $d = 2$ at these temperatures, corresponding to the entropy being dominated by the photons, the condition $\lambda_{\mathrm{T}} = l$ determines the temperature T_{ps} of the photosphere.

$$\sqrt{\frac{T_{\mathrm{ps}}}{m_{\mathrm{e}}}} \exp(m_{\mathrm{e}}/T_{\mathrm{ps}}) = \frac{T_{\mathrm{h}}}{9.2\,\mathrm{GeV}} \tag{12}$$

Thus the photosphere temperature decreases as the black hole temperature increases, roughly in a logarithmic manner. Requiring $T_{\mathrm{ps}} < 100$ keV means that $T_{\mathrm{h}} > 680$ GeV. For black hole temperatures greater than about 700 GeV essentially all of the luminosity is in the form of photons, with the gravitons emitted directly without rescattering, and the neutrinos having lost thermal contact at some very small radius and correspondingly high temperature.

The Lorentz gamma factor at the photosphere is determined by eq. (7) to be $\gamma_{\mathrm{ps}} = T_{\mathrm{h}}/T_{\mathrm{ps}}$. When the black hole temperature is 1 TeV the temperature of the photosphere is 100 keV and $\gamma_{\mathrm{ps}} = 10^7$. Let us now calculate the radiation from a surface moving at relativistic velocity. The phase space density f is invariant. Hence

$$f(\mathbf{p}, \mathbf{x}, t) = f(\mathbf{p}', \mathbf{x}', t') = \frac{1}{\exp(E'/T_{\mathrm{ps}}) - 1} \tag{13}$$

where the unprimed variables refer to the black hole rest frame and the primed variables refer to the rest frame of the fluid. We desire the flux in the black hole rest frame. Using $E' = \gamma_{\mathrm{ps}} E(1 - v_{\mathrm{ps}} \cos\theta)$ the spectral flux is

$$\frac{dF_\gamma}{dE} = \frac{E^3}{2\pi^2} \int_0^1 d(\cos\theta)\, \cos\theta\, f(E, \cos\theta)\,. \tag{14}$$

The denominator of f may be expanded in a Taylor series and integrated term by term. The resulting series cannot be summed in terms of elementary functions. However, in the

limit that $\gamma_{\rm ps} \gg 1$ and $E \gg T_{\rm ps}/\gamma_{\rm ps}$ (a very small energy indeed!) one gets

$$\frac{dF_\gamma}{dE} = -\frac{E^2 T_{\rm h}}{2\pi^2\gamma_{\rm ps}^2} \ln\left[1 - \exp\left(-E/2T_{\rm h}\right)\right] . \tag{15}$$

Integration over all energy gives the total flux (energy/area/time).

$$F_\gamma = \frac{8\pi^2}{90}\gamma_{\rm ps}^2 T_{\rm ps}^4 = \frac{8\pi^2}{90}\frac{T_{\rm h}^4}{\gamma_{\rm ps}^2} \tag{16}$$

One may also integrate eq. (13) exactly to get the total flux; in the limit $\gamma_{\rm ps} \gg 1$ one obtains the same result. The corresponding expressions for each flavor of neutrino are

$$\begin{aligned} \frac{dF_\nu}{dE} &= \frac{E^2 T_{\rm h}}{2\pi^2\gamma_\nu^2} \ln\left[1 + \exp\left(-E/2T_{\rm h}\right)\right] , \\ F_\nu &= \frac{7\pi^2}{90}\gamma_\nu^2 T_\nu^4 = \frac{7\pi^2}{90}\frac{T_{\rm h}^4}{\gamma_\nu^2} . \end{aligned} \tag{17}$$

The temperature and Lorentz gamma-factor at which the neutrinos lose thermal contact will be addressed in another paper.

Equating the total luminosity of the black hole, excluding that carried by gravitons and neutrinos, with $4\pi R_{\rm ps}^2 F_\gamma$ determines the radius of the photosphere.

$$R_{\rm ps}^2 = \frac{180}{\pi}\frac{\alpha_{\rm eff}}{T_{\rm ps}^2} \tag{18}$$

Here $\alpha_{\rm eff} = \frac{406}{435}\alpha$ with α evaluated at $T_{\rm h} > 100$ GeV: the standard model degrees of freedom are assumed to hold all the way up to the Planck temperature. Starting with a black hole of temperature $T_{\rm h} > 100$ GeV the time it takes to evaporate/explode is

$$\Delta t = \frac{m_{\rm P}^2}{3\alpha(8\pi T_{\rm h})^3} . \tag{19}$$

This is also the characteristic time scale for the rate of change of the luminosity of a black hole with temperature $T_{\rm h}$. It is many orders of magnitude greater than the time it takes a light signal to travel from the Schwarzschild radius out to the photosphere, thus justifying the quasi-stationary assumption. A black hole with temperature 1 TeV has a Schwarzschild radius of 1.57×10^{-5} fm, its photosphere is at $R_{\rm ps} = 970$ fm, and has 464 seconds to live.

Knowing how the temperature, flow velocity and radius of the photosphere evolve with time allows us to determine the energy distribution of photons emitted during the final minutes of the black hole. Integrating from $t = -\Delta t$ to $t = 0$ gives

$$\frac{dN_\gamma}{dE} = -\frac{45}{4\pi^5}\frac{\alpha_{\rm eff}}{\alpha}\frac{m_{\rm P}^2}{E^3}\int_0^{E/2T_{\rm h}} dx\, x^4 \,\ln(1 - e^{-x}) . \tag{20}$$

Both this integrated distribution and the instantaneous one, $4\pi R_{\rm ps}^2 dF_\gamma/dE$, are insensitive to the precise location of the photosphere. In the limit $E \gg 2T_{\rm h}$ eq. (20) becomes

$$\frac{dN_\gamma}{dE} = \frac{4\pi}{15}\frac{m_{\rm P}^2}{E^3}\,. \tag{21}$$

This energy distribution should be valid for any primordial black hole for photon energies much larger than 1 TeV = 10^{12} eV.

The hot shell of matter surrounding a primordial black hole provides a theoretical testing ground rivaled only by the big bang itself. In addition to the questions already raised, one may contemplate baryon number violation at high temperature and how physics beyond the standard model might be important in the last few minutes in the life of a primordial black hole. Experimental discovery of exploding black holes will be one of the great challenges in the new millenium.

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References

[1] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).

[2] B. J. Carr and J. H. MacGibbon, Phys. Rep. **307**, 141 (1998).

[3] Particle Data Group: *Review of Particle Physics*, Eur. Phys. J. **C3**, 1 (1998).

[4] A. F. Heckler, Phys. Rev. D **55**, 480 (1997); Phys. Rev. Lett. **78**, 3430 (1997).

[5] J. Oliensis and C. T. Hill, Phys. Lett. **B143**, 92 (1984); J. H. MacGibbon and B. R. Webber, Phys. Rev. D **41**, 3052 (1990); J. H. MacGibbon and B. J. Carr, Astrophys. J. **371**, 447 (1991); F. Halzen, E. Zas, J. H. MacGibbon and T. C. Weekes, Nature (London) **353**, 807 (1991).

[6] K. S. Thorne and R. A. Flammang, Mon. Not. R. Astr. Soc. **194**, 475 (1981); R. A. Flammang, *ibid.* **199**, 833 (1982).

[7] B. Paczyǹski, Astrophys. J. **308**, L43 (1986).

[8] D. N. Page, Phys. Rev. D **13**, 198 (1976).

[9] N. Sanchez, Phys. Rev. D **18**, 1030 (1978). The coefficient was extracted from figure 5 of this paper to within about 5% accuracy.

[10] One may usefully contemplate whether the helicity-0 component of the W and Z bosons below the symmetry restoring temperature is to be treated as the third component of a free massive vector boson or as a free massive scalar particle on account of the Higgs mechanism. If the vexing problem of quark/gluon versus hadron production has not attracted the curiosity of the reader this ought to.

[11] K. Kinder-Geiger, Phys. Rep. **258**, 237 (1995).

[12] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989).

[13] M. E. Carrington and J. I. Kapusta, Phys. Rev. D **47**, 5304 (1993).

[14] See, for example, the proceedings of the Quark Matter series of international conferences, the most recent in print being: *Proceedings of Quark Matter '97*, Nucl. Phys. **A638**, (1998).

[15] D. Mihalas, *Stellar Atmospheres*, 2nd ed. (Freeman, San Francisco, 1978).